

N90-19896
TMX-53866

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NASA TM X-53866

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Z TRANSFORM AND THE USE OF THE DIGITAL DIFFERENTIAL
ANALYZER AS A PERIPHERAL DEVICE TO A GENERAL
PURPOSE COMPUTER

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August 12, 1969

NASA

*George C. Marshall Space Flight Center
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TECHNICAL REPORT STANDARD TITLE PAGE

1. Report No. TMX-53866	2. Government Accession No.	3. Recipient's Catalog No.	
4. Title and Subtitle Z Transform and the Use of the Digital Differential Analyzer as a Peripheral Device to a General Purpose Computer		5. Report Date August 12, 1969	6. Performing Organization Code
7. Author(s) Donald Nalley		8. Performing Organization Report No.	
9. Performing Organization Name and Address George C. Marshall Space Flight Center Marshall Space Flight Center, Alabama 35812		10. Work Unit No.	11. Contract or Grant No.
12. Sponsoring Agency Name and Address		13. Type of Report and Period Covered Technical Memorandum	
14. Sponsoring Agency Code			
15. Supplementary Notes <p>Prepared by Computation Laboratory, Science and Engineering Directorate</p>			
16. Abstract This work was undertaken to determine the feasibility of the Digital Differential Analyzer (DDA) as a peripheral device; that is, a computer system programmed in Fortran IV or some other standard language in which certain manipulations could be accomplished on the DDA by calling a subroutine. In this study, consideration was given to Large Scale Integrator Circuits (LSI), because it was felt that suitable LSI would make the DDA a more feasible device.			
From the study, it appears that the DDA as a standard peripheral device with automatic patching is indeed feasible. However, implementation with LSI is questionable presently, but this may become feasible soon.			
A portion of time was allotted to the study of a control system analysis program using Z transform calculus. This work was performed to supplement the DDA work by extending the application of Z transform calculus to the analysis of the DDA. An analysis of IBM's control system analysis program was made, and several example problems were programmed using this program. The results and an explanation of these problems are given. (Cont'd)			
17. Key Words Digital Differential Analyzer, Peripheral Device, Large Scale Integrated Circuits, Z Transform	18. Distribution Statement <i>H. W. Zeanah</i>		
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 31	22. Price \$3.00

The major advantage of the control system analysis program is that it can be used to derive difference equations, which are necessary to accomplish real-time flight simulation by digital computer techniques. It is felt that the program may be used in further analysis of the DDA.

ACKNOWLEDGMENTS

The inspiration for this work came from Dr. H. Trauboth's suggested research assignment entitled "Application of the Digital Differential Analyzer as Peripheral Device to a General Purpose Computer." Appreciation is expressed also to Mr. Hugh Zeanah for the preliminary work performed in the area and his assistance, guidance, and encouragement during the study.

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Z TRANSFORM AND THE USE OF THE DIGITAL DIFFERENTIAL ANALYZER AS A PERIPHERAL DEVICE TO A GENERAL PURPOSE COMPUTER

SUMMARY

This work was undertaken to determine the feasibility of the Digital Differential Analyzer (DDA) as a peripheral device; that is, a computer system programmed in Fortran IV or some other standard language in which certain manipulations could be accomplished on the DDA by calling a subroutine. In this study, consideration was given to Large Scale Integrator Circuits (LSI), because it was felt that suitable LSI would make the DDA a more feasible device.

From the study, it appears that the DDA as a standard peripheral device with automatic patching is indeed feasible. However, implementation with LSI is questionable presently, but this may become feasible soon.

A portion of time was allotted to the study of a control system analysis program using Z transform calculus. This work was performed to supplement the DDA work by extending the application of Z transform calculus to the analysis of the DDA. An analysis of IBM's control system analysis program was made, and several example problems were programmed using this program. The results and an explanation of these problems are given.

The major advantage of the control system analysis program is that it can be used to derive difference equations, which are necessary to accomplish real-time flight simulation by digital computer techniques. It is felt that the program may be used in further analysis of the DDA.

INTRODUCTION

In the study of analog digital and hybrid computation, the author became interested in various approaches to the problem of solving differential equations. In reading about the Digital Differential Analyzer (DDA), it occurred to the author that the speed of the DDA in making calculations would be a tremendous advantage if it could be used along with the digital computer, or possibly as a very good competitor with the general purpose digital computer.

For the past year at Clemson University, the investigator has been teaching a course to seniors and first year graduate students in "Analog Digital and Hybrid Computation." In this experience, many problems were encountered; for example, patching of the Analog Computer, no hands on capability with digital, etc. From these problems, the feeling developed that the DDA may be suitable for alleviating some of the problems with computation.

RESEARCH OBJECTIVES

The following items were the objectives of this research study:

1. To become familiar with the TRICE DDA.
2. To search the literature to determine the state of the art of the DDA.
3. To analyze the TRICE DDA using the Z transform calculus.
4. To determine the feasibility of implementing a DDA using Large Scale Integrator (LSI) circuits.
5. To define, design, and implement an interface for the DDA.
6. To determine the numerical method best suited for the DDA and to develop the necessary software.

RESULTS OF STUDY

Item 1 of the research objectives was accomplished partially by working on the hardware with Mr. Hugh Zeanah. A few problems were programmed and solved in order to obtain a basic working knowledge of the TRICE. More familiarity with the DDA was gained while studying the literature during the literature search, listed as item 2 of the research objectives.

After obtaining a reasonable feeling for the state of the art, thought was given to some means of analyzing the TRICE DDA. IBM's control system analysis program (7090-MA-OIX) was already in existence and had previously been studied by Mr. Zeanah. This program has the capability to change control system relations from the S domain to the Z domain; and it was felt that this program would be helpful in analyzing the TRICE DDA. Therefore, effort was exerted to get the control system analysis program working to obtain the Z transform of several control problems. The results are given in the Appendix.

Figure A-1 of the Appendix shows the diagram of the system reduced to the S domain notation. Figure A-2 gives the necessary input data for obtaining a Z transform and root locus are given in Figure A-3. The printed results are presented in four different forms. The last three forms are variations of the format given in Reference 1, page 14.

It is noticed that one value in the B matrix does not agree with that of the author's calculations. This was checked several times, however, and apparently there is a mistake in Reference 1.

The first set of data is for 20 iterations, that is, the program carries out 41 terms (a=20 of the series for $G^*(S)$).

$$G^*(S) = \frac{1}{T} \sum_{n=-a}^{a} G(S+in\frac{2\pi}{T})$$

This relation represents the transfer function describing the output of a "sampler" that is sampling the impulse response of a continuous system $G(S)$, where n is the number of samples and T is the time between samples. If a different number of terms is desired, the value of a is inserted in columns 40 through 42 of the control card. The program automatically carries out 20 iterations if columns 40 through 42 are left blank.

The second set of data is the same as the first except that the B matrix is left out. The B matrix consists of coefficients for the set of simultaneous equations used in calculating a Z transform. The B matrix is printed only if a -1 is set in field 17 (columns 49 through 51) of the control card.

The third set of data is for 9 iterations (19 terms in the series for $G^*(S)$). The B matrix is also printed out.

The fourth set of data is for 9 iterations. Here, only the Z transform, and not the root locus points from it, is computed. This is done by setting n=0 into field 16 (columns 46 through 48) where n should be equal to the degree of the resulting Z transform denominator polynomial. (For details of the program, see Reference 1.)

In order to further experiment with the control system analysis program, example problems were programmed [2, page 442]. The computer results compare favorably with Tou's results. (Refer to the Appendix.)

Harold Scofield of R-ASTR-FG was contacted concerning transforming control system problems. It was discovered that his group had experimented with obtaining Z transforms, but the group had not been able to carry the work as far as desired. It is believed that this program will be helpful in changing almost any of their control system problems into Z transform problems, and then into a difference equation to study the systems with a digital computer. It is suggested that more work be put into this effort, since the program will be helpful to R-ASTR-FG and will be a useful tool in analyzing the TRICE.

In establishing the feasibility of implementing a DDA using LSI, it was necessary to get some idea of the state of the art of LSI.

The present number of circuits obtainable from a pad leaves something to be desired. During fabrication, a portion of a pad may become defective and destroy circuit capability. If a portion of the circuit becomes defective there is presently no means of bypassing this defective portion, thus causing the entire board to become defective. It is also very difficult to build the LSI either automatically or by hand. Handwork is especially poor because of the small device involved.

Generally, a custom integrated chip of LSI is very expensive but may become cheaper when the manufacturing techniques are improved.

LSI is very suitable where a number of repeated circuits exist.

According to the Teledyne Systems Company of Northridge, California, a DDA implemented with LSI is feasible. They report of a Teledyne Electrically Alterable Digital Differential Analyzer (TEADDA), which is a parallel word, parallel computation Digital Differential Analyzer (DDA) with a speed of 1 000 000 iterations (complete incremental solutions) per second and a resolution of one part per million. The TEADDA is programmed by interconnecting the computational elements electrically, controlled from a general purpose digital computer, tape reader, or other peripheral equipment.

The TEADDA is implemented with Teledyne's hybrid LSI Micro-Electronic Modular Assembly (MEMA) technique to achieve the high speed operation. High speed performance is achieved as a result of the small computer size and the MEMA being packaged so that short connections are used. The significant features of the TEADDA are as follows:

1. 1-MHz iteration rate
2. 20-bit resolution (one part per million)
3. "Electronic Patchboard" for completely automatic programming
4. 100 percent interconnectivity between 64 integrators (256 programmable connections for each increment)
5. Extrapolative trapezoidal integration
6. Automatic register length selection to vary the incremental weighting over a range of 2^{16} (approximately 65 000)
7. Multiple incremental inputs to each integrator
8. Ternary incremental communication
9. Two's complement negative number representation
10. MEMA packaging
11. High reliability
12. Low cost

For more details on the TEADDA, see Reference 3, pages 161-169.

Items 5 and 6 of the objectives are being studied further. Also, more study is needed to firmly establish the feasibility of an LSI implemented DDA.

CONCLUSIONS AND RECOMMENDATIONS

It has been established that for some purposes, the DDA is more effective than a general purpose digital computer or an analog computer. It was discovered that digital computers and analog computers are extremely unwieldy in comparison with the DDA for some uses. In one case, it was found that solving an integral equation on a general purpose machine took 2 weeks, whereas on the DDA the time was 2 hours.

There are real advantages to be gained in connecting the DDA to a high speed digital computer, since the integration operation on a DDA is comparable with the integration speed on a digital computer.

When considering complexity and nonlinearity, the DDA is a very effective machine since the amount of equipment does not increase in proportion to the complexity of the problem.

RECOMMENDATIONS FOR FURTHER STUDY

It is recommended that the following further studies be performed to supplement and advance the present work.

1. Continue to study control problems with the control system analysis program. With this program, it is expected that the Z transform of systems expressed in S transform can be derived. From the Z transform a difference equation can be derived that can be easily programmed on the digital computer. This technique can be used to further study the TRICE DDA.
2. When sufficient information is obtained from item 1 above, modify the program or develop a program so that Z transforms can be automatically obtained from input and output pulse patterns. This would allow transforms to be derived for more complicated interconnections of TRICE elements.
3. Make further studies of the availability and use of LSI to determine whether they are suitable for developing a new DDA.
4. Continue work on items 5 and 6 of the Research Objectives. This should become a three or four man effort in order to be completed in about 1 year.

APPENDIX

COMPUTER RESULTS

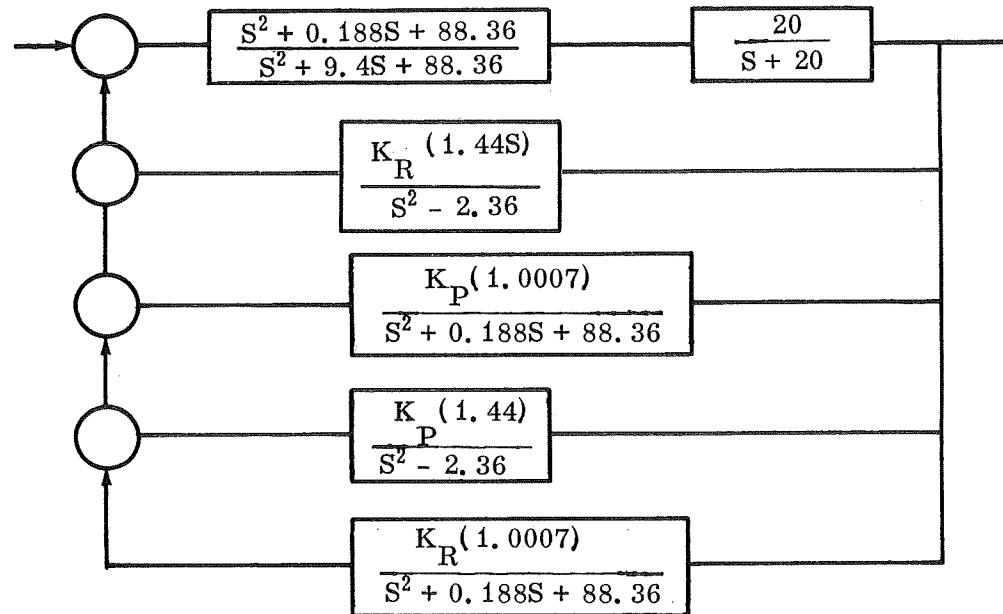


Figure A-1. System diagram, example 1.

With the ratio of K_R/K_P held constant at 0.6, the three runs will be made in this order: The first will be for the nominal values of Figure A-1, or perfect cancellation. The second will be for the case of the poles reduced in frequency by 20 percent. The third will be for the case with their frequency increased by 20 percent. Thus, for the first part of this triple run, with $K_P = 1$, having (arbitrarily) chosen to begin with the bottom feedback loop and work upward, we may work out an A-vector and M-vector sheet as follows[1]:

Card #	1	54	38	4	0	-14	3	5	1		-2		20		5	-1
2. Report Heading																
3	-0.5	0.5	2.		0.		6.		12.		.2					
4	-1.536	1.		1.536	1.		20.	1.			88.36	9.45	1.			
5	-1.007	88.36	0.1885	1.		1.44		-2.36		0.		1.	0.			
6	-0.6042	88.36	0.1885	1.		0.		0.864		-2.36	0.		1.			
7	20.	88.36	0.1885	1.		-1.		1.			20.	1.	0.			
8	88.365	9.45	1.		0.		1.		0.		9.	-1.	-5.			
9	4	1	1	2	0	1	0	1	2	1	0	1	2	1	1	3
10	0	2	-1	3	1	3	-1	1	-17	2	-14	-17	1	-16		
11 Z TRANSFORM (20 ITERATIONS NO B MATRIX).																
12	54	38	4	0	-14	3	5	1			-2	20	5	0		
13 ROOT LOCUS WITH CALCULATED Z TRANSFORM 9 ITERATIONS																
14	54	38	4	0	-14	3	5	1			-2	9	5	-1		
15 Z TRANSFORM ONLY (9 ITERATIONS)																
16	54	38	4	0	-14	3	5	1			-2	20	-5	-1		

Figure A-2. Data for example 1.

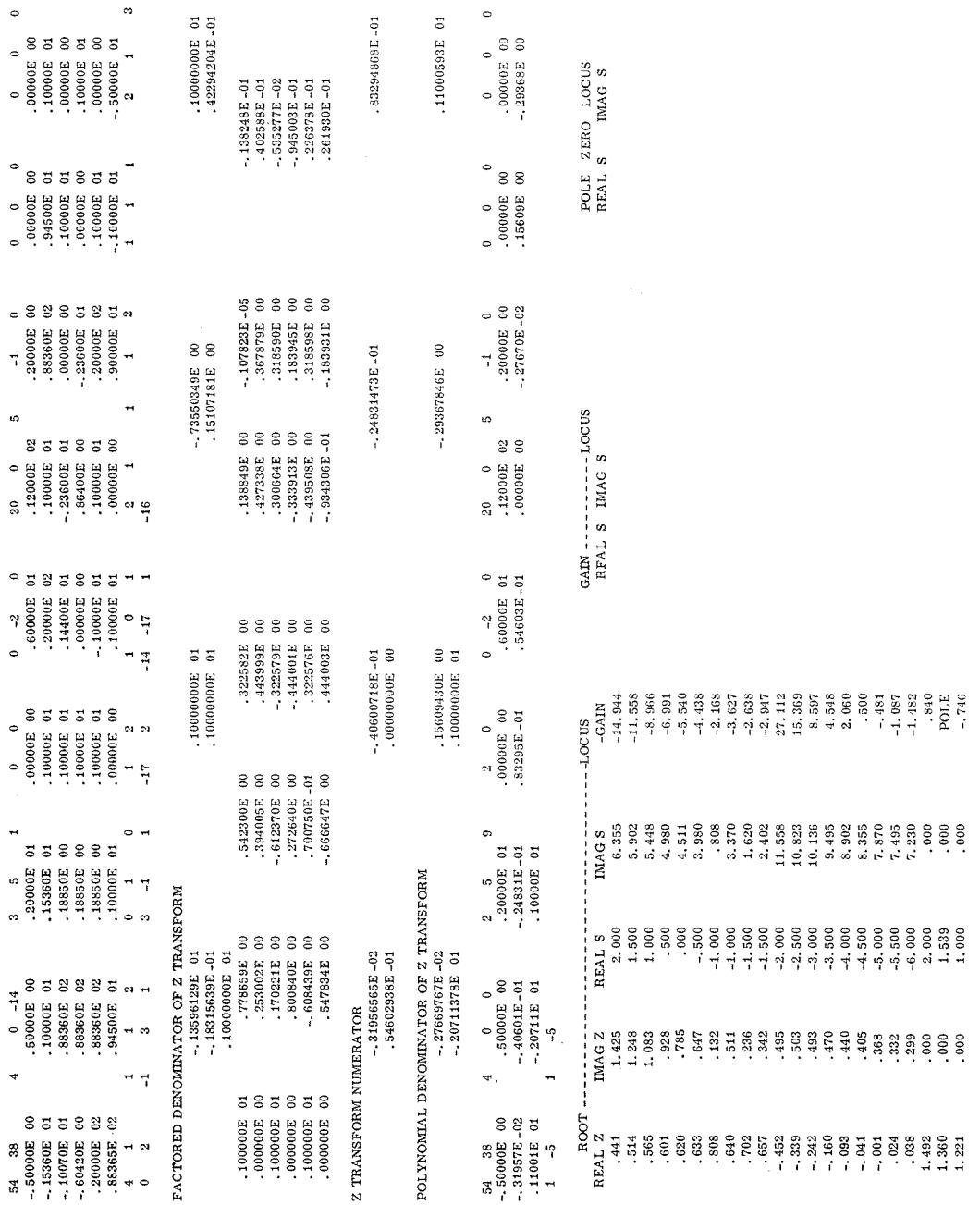


Figure A-3. Root locus of a continuous system with calculated Z transform (20 iterations). (sheet 1 of 7)

1.105	.000	.500	.000	-1.256
1.000	.000	.000	.000	-1.609
.905	.000	-.500	.000	-1.809
.819	.000	-1.000	.000	-1.797
.741	.000	-1.500	.000	-.510
.670	.000	-2.000	.000	-3.376
.607	.000	-2.500	.000	-2.833
.549	.000	-3.000	.000	-2.682
.497	.000	-3.500	.000	-2.587
.449	.000	-4.000	.000	-2.505
.407	.000	-4.500	.000	-2.429
.368	.000	-5.000	.000	-2.354
.333	.000	-5.500	.000	-2.283
.301	.000	-6.000	.000	-2.214
.279	.084	-.852	.500	-1.978
.784	.159	-1.117	1.000	-2.300
.716	.221	-1.445	1.500	-2.588
.675	.285	-1.555	2.000	-2.780
.653	.357	-1.477	2.500	-2.994
.644	.440	-1.242	3.000	-3.302
.639	.538	-.898	3.500	-3.770
.633	.652	-.477	4.000	-4.477
.621	.782	-.008	4.500	-5.517
.600	.934	.523	5.000	-7.068
.560	1.100	1.055	5.500	-9.220
.501	1.288	1.617	6.000	-12.250
.024	.333	-5.492	7.500	-1.080
-.011	.378	-4.867	8.000	-.256
-.054	.415	-4.352	8.500	.874
-.104	.445	-3.914	9.000	2.408
-.161	.471	-3.492	9.500	4.582
-.224	.489	-3.102	10.000	7.594
-.293	.500	-2.727	10.500	11.815
-.365	.502	-2.383	11.000	17.639
-.443	.496	-2.039	11.500	25.950
-.522	.478	-1.727	12.000	37.519

A VECTOR COUNT = 50 M VECTOR COUNT = 31

Figure A-3. Root locus of a continuous system with calculated Z transform (20 iterations). (sheet 2 of 7)

```

34 38 4 0-14   3 5 1   -2 20   5 0
      0  -14   2  5   9   2  0   0   0  -2  0
      .5000E 00  .2000E 01  .0000E 00  .6000E 01  .2000E 02
      -.5000E 00  -.15360E 01  .15360E 01  .2000E 02  -.1000E 01
      -.15360E 01  .1000E 01  .1000E 01  .1000E 02  .1000E 01
      -.10070E 01  .88360E 02  .88360E 02  .1440E 01  .1440E 01
      -.60420E 00  .88360E 02  .88360E 02  .18850E 00  .18850E 00
      .20000E 02  .88360E 02  .88360E 02  .18850E 00  .18850E 00
      .88365E 02  .94300E 01  .10000E 01  .10000E 01  .10000E 01
      4  1     1  2   0   1  0   1  2   0   1   1  2   1   1  2
      0  2     -1  3  1   3  -1  1   -17  2   -14  -17  1   -16
      FACTORED DENOMINATOR OF Z TRANSFORM
      -.13556129E 01  .10000000E 01  .10000000E 01  .10000000E 01
      -.132156329E 01  .10000000E 01  .10000000E 01  .10000000E 01
      Z TRANSFORM NUMERATOR
      -.339563655E -02  .40600718E -01  .00000000E 00  .10000000E 00
      .546023838E -01  .150718E 00  .150718E 00  .10000000E 00
      POLYNOMIAL DENOMINATOR OF Z TRANSFORM
      -.27659767E -02  .15609430E 00  .100000000E 01  .100000000E 01
      54  38   4     0   0   2  5   9   2  0   0   0  -2  0
      .50000E 00  .50000E 00  .20000E 01  .00000E 00  .60000E 01  .20000E 02
      -.50000E 00  -.40600E -01  -.24631E -01  .88359E -01  .40600E -01  .27670E -02
      -.31397E -02  .601  .928  .500  .000  .4511  .5410
      .110001E 01  -.20711E 01  .10000E 01  .10000E 01  .10000E 01
      1  -6     1  -5
      ROOT ----- LOCUS ----- GAIN ----- POLE ZERO LOCUS
      REAL Z   IMAG Z   REAL S   IMAG S   GAIN   REAL S   IMAG S   REAL S   IMAG S
      .441  1.425  2.000  6.355  -.14.944
      .514  1.238  1.500  5.302  -.11.558
      .565  1.083  1.000  5.448  -.8.966
      .601  .928  .500  4.380  -.6.391
      .630  .785  .000  4.511  -.5.540
      .633  .647  -.500  3.380  -.4.438
      .808  .132  -.1.000  -.508  -.2.438
      .640  .511  -.1.000  3.370  -.3.627
      .702  .236  -.1.500  1.620  -.2.638
      .637  .342  -.1.500  2.402  -.2.947
      .452  .495  -.2.000  11.658  .27.112
      .339  .503  -.2.500  10.823  .15.369
      .242  .493  -.3.000  10.136  .8.537
      .160  .470  -.3.500  3.495  .4.548
      .033  .440  -.4.000  8.302  .2.060
      .041  .405  -.4.500  8.355  .500
      .001  .368  -.5.000  7.870  .-1.481
      .024  .332  -.5.500  7.495  .-1.087
      .038  .289  -.6.000  7.230  .-1.182
      1.492  .000  2.000  .000  .840
      1.360  .000  1.539  .000  .POLE
      1.221  .000  1.000  .000  .-746
      1.105  .000  .500  .000  .-1.256
      1.000  .000  .000  .000  .-1.609
      .905  .000  -.5.000  .000  .-1.869
      .819  .000  -.1.000  .000  .-1.797
      .741  .000  -.1.500  .000  .-510

```

Figure A-3. Root Locus of a continuous system with calculated Z transform (20 iterations). (sheet 3 of 7)

.670	.000	-2. 500	-2. 833
.549	.000	-3. 000	-2. 682
.497	.000	-3. 500	-2. 587
.449	.000	-4. 000	-2. 505
.407	.000	-4. 500	-2. 429
.368	.000	-5. 000	-2. 354
.333	.000	-5. 500	-2. 283
.301	.000	-6. 000	-2. 214
.289	.084	-7. 852	-1. 978
.784	.159	-1. 117	-2. 300
.716	.221	-1. 445	-2. 588
.675	.285	-1. 555	-2. 780
.653	.357	-1. 477	-2. 994
.644	.440	-1. 242	-3. 302
.639	.538	-898	-3. 770
.633	.652	-477	-4. 477
.621	.782	-008	-5. 517
.600	.934	.523	-7. 068
.560	1. 100	1. 055	-9. 220
.501	1. 288	1. 617	-12. 250
.024	.333	-5. 492	-1. 080
-.011	.378	-4. 867	-2. 256
-.054	.415	-4. 352	.874
-.104	.445	-3. 914	2. 408
-.161	.471	-3. 492	4. 582
-.224	.489	-3. 102	7. 594
-.293	.500	-2. 727	11. 815
-.365	.502	-2. 383	17. 639
-.443	.496	-2. 039	25. 950
-.522	.478	-1. 727	37. 519

A VECTOR COUNT = 51

M VECTOR COUNT = 33

Figure A-3. Root locus of a continuous system with calculated Z transform (20 iterations). (sheet 4 of 7)

FACTORED DENOMINATOR OF Z TRANSFORM					
- .13586129E 01					
- .18315639E -01					
.100000E 01	.772653E 00	.542300E 00	.322582E 00	.138849E 00	-.122960E -05
.000000E 00	.253002E 00	.394005E 00	.443399E 00	.427388E 00	.402588E -01
.100000E 01	.170221E 00	.612370E 00	.322579E 00	.300644E 00	-.535307E -02
.000000E 00	.500849E 00	.272640E 00	.444001E 00	.335913E 00	.444008E -01
.100000E 01	-.604549E 00	.700750E -01	.322576E 00	.435088E 00	.261937E -01
.000000E 00	.547834E 00	-.666647E 00	.444003E 00	-.934306E -01	-.182331E 00
Z TRANSFORM NUMERATOR					
- .31963833E -02					
*.54602849E -01					
POLYNOMIAL DENOMINATOR OF Z TRANSFORM					
- .27659767E -02					
- .20711378E 01					
.54 38	4	6	5	9	
-.50000E 00	.50000E 00	.20000E 01	.20000E 00	.60000E 01	.12000E 02
-.31964E -02	-.40600E -01	-.24833E -01	.20000E 01	.60000E 01	.12000E 02
.11001E 01	-.20711E 01	-.10000E 01	.83297E -01	.54603E -01	.00000E 00
1 -5	1	-5			
ROOT					
REAL Z	IMAG Z	REAL S	IMAG S	GAIN	LOCUS
.441	1.425	2.000	6.355	-.14.944	POLE ZERO LOCUS
.514	1.248	1.500	5.992	-.11.538	REAL S IMAG S
.565	1.083	1.000	5.448	-.8.966	
.601	.928	.500	4.980	-.6.991	
.620	.785	.000	4.511	-.5.540	
.633	.647	-.500	3.980	-.4.458	
.808	.132	-.1.000	.808	-.2.168	
.640	.511	-.1.000	.3.370	-.3.627	
.702	.236	-.1.500	1.620	-.2.638	
.657	.342	-.1.500	2.492	-.2.947	
.452	.495	-.2.000	11.558	-.481	
-.339	.503	-.2.500	10.823	15.370	
-.242	.438	-.3.000	10.136	8.597	
-.160	.470	-.3.500	9.495	4.548	
-.093	.440	-.4.000	8.902	2.060	
-.041	.405	-.4.500	8.355	.500	
-.004	.368	-.5.000	7.870	-.481	
.024	.332	-.5.500	7.495	-.1.087	
.038	.299	-.6.000	7.230	-.1.482	
1.492	.000	2.000	.000	.840	POLE
1.360	.000	1.539	.000	-.746	
1.221	.000	1.000	.000		

Figure A-3. Root locus of a continuous system with calculated Z transform (20 iterations). (sheet 5 of 7)

1.105	.000	.500	.000	-1.256
1.000	.000	.000	.000	-1.609
.905	.000	-.500	.000	-1.809
.819	.000	-1.000	.000	-1.797
.741	.000	-1.500	.000	-.510
.670	.000	-2.000	.000	-3.376
.607	.000	-2.500	.000	-2.833
.549	.000	-3.000	.000	-2.682
.497	.000	-3.500	.000	-2.587
.449	.000	-4.000	.000	-2.505
.407	.000	-4.500	.000	-2.429
.368	.000	-5.000	.000	-2.354
.333	.000	-5.500	.000	-2.283
.301	.000	-6.000	.000	-2.214
.289	.084	-.852	.500	-1.978
.784	.159	-1.117	1.000	-2.300
.716	.221	-1.445	1.500	-2.588
.675	.285	-1.555	2.000	-2.780
.653	.357	-1.477	2.500	-2.994
.644	.440	-1.242	3.000	-3.302
.639	.538	-.898	3.500	-3.770
.633	.652	-.477	4.000	-4.477
.621	.782	-.008	4.500	-5.517
.600	.934	.523	5.000	-7.068
.560	1.100	1.055	5.500	-9.220
.501	1.288	1.617	6.000	-12.250
.024	.333	-5.492	7.500	-1.080
-.011	.378	-4.867	8.000	-.256
-.054	.415	-4.352	8.500	.874
-.104	.445	-3.914	9.000	2.408
-.161	.471	-3.492	9.500	4.582
-.224	.489	-3.102	10.000	7.594
-.293	.500	-2.727	10.500	11.815
-.365	.502	-2.383	11.000	17.639
-.443	.496	-2.039	11.500	25.950
-.522	.478	-1.727	12.000	37.519

A VECTOR COUNT = 50 M VECTOR COUNT = 31

Figure A-3. Root locus of a continuous system with calculated Z transform (20 iterations). (sheet 6 of 7)

Figure A-3. Root locus of a continuous system with calculated Z transform (20 iterations). (sheet 7 of 7)

Shown in Figure A-4 is the block diagram of an error-sampled feedback control system. The sampling period is 0.1 second. A zero order hold is used and the transfer function of the controlled system is

$$G_S(S) = \frac{30}{S(1 + 0.1S)}$$

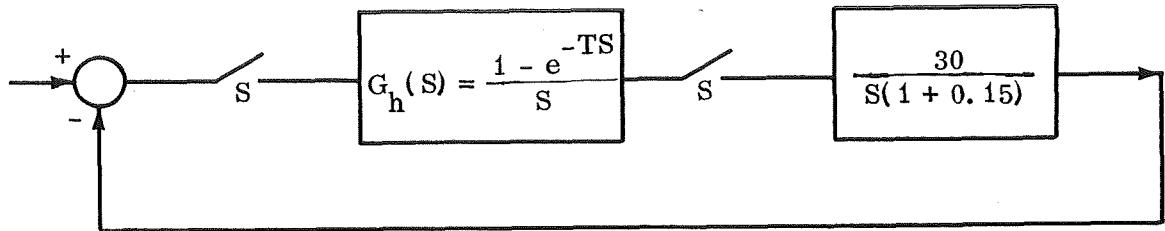


Figure A-4. System description, example 2.

The forward transfer function is

$$G(S) = \frac{30(1 - e^{-TS})}{S^2(1 + 0.1S)}$$

The Z transform associated with G(S) is found to be

$$G(Z) = \frac{1.104(Z + 0.718)}{(Z-1)(Z - 0.368)}$$

The computer results (Fig. A-5) compare identically.

$$G(Z) = \frac{1.1036365Z + 0.7972486}{Z^2 - 1.3678794Z + 0.36787944}$$

Card #	Input Data Card														
1	28	10	1	0	-11	3	2	1	1					-2	
2	-, 5	.	. 5		2,		0,		6,	12,	.	1			
3	10.		1.		-1.		1.		300.	-1.	1.	0.		0.	
4	10.		1.		0.		1.								
5	2	1	-1	0	2	0	-1	2	3	-1					

Computer Results For Example 2

FACTORED DENOMINATOR OF Z TRANSFORM

-.36787944E 00 .10000000E 01

-.10000000E 01 .10000000E 01

Z TRANSFORM NUMERATOR

.79272486E 00 .11036365E 01

.00000000E 00

POLYNOMIAL DENOMINATOR OF Z TRANSFORM

.36787944E 00 -.13678794E 01

.10000000E 01

Figure A-5. Data for example 2.

The following example demonstrates the ability of the control system analysis program to transform a ratio of polynomial, part of which may be in S and part in Z, into the Z domain. The block diagram of the system is given in Figure A-6. Note that this is Example 2 modified by additional networks that are already expressed in Z notation.

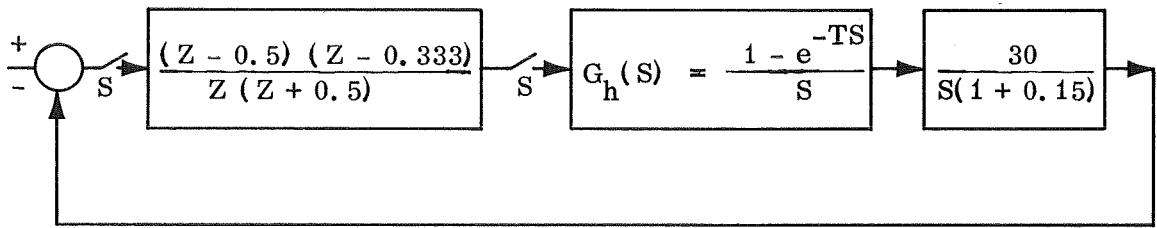


Figure A-6. System description, example 3.

$$G = \frac{30 (1 - e^{-Ts}) (Z - 0.5) (Z - 0.333)}{s(s) (1 + 0.15) Z(Z + 0.5)}$$

$$G = \frac{300 (1 - Z^{-1}) (Z - 0.5) (Z - 0.333)}{s(s) (s + 10)(Z) (Z + 0.5)}$$

$$G = \frac{300 (Z-1) / Z (Z - 0.5) (Z - 0.333)}{Z (Z + 0.5) (s^2) (s + 10)}$$

The zero order hold is not considered as part of the system poles.
Hence the system poles are:

$$\frac{(Z+0) (Z+0.5)(Z-1) (s+10)}{1}$$

The pole at the origin, S , is entered as $Z-1$.

The parameters are now entered for the entire system expression:

$$G = \frac{(Z-1) (Z-0.5) (Z-0.333) (300)}{(Z^3 + 0.5Z^2 + 0Z + 0) (s^3 + 10s^2 + 0s + 0)}$$

From the two equations immediately above, the input data are obtained and are given in Figure A-7.

Card #

1	32	14	1	0	-11	3	4	1	1						-4		
2	-.5		.5		2.		0.		6.	12.		.1					
3	10.		1.		-1.		1.		.5	1.	0.	1.	300.				
4	-.333		1.		-.5		1.		-1.	1.	0.	0.	10.				
5	1.		0.		0.		.5		1.								
6	4	1	-1	-1	-1	0	4	0	-1	-1	-1	2	3	-3			

Computer Results For Example 3

Factored Denominator of Z Transform

-. 36787944E00	. 10000000E01
-. 10000000E01	. 10000000E01
. 50000000E00	. 10000000E01
. 00000000E00	. 10000000E01

Z Transform Numerator

. 13199240E00	-. 47659026E00
-. 12659967E00	. 11036323E01
. 00000000E00	

Polynomial Denominator of Z Transform

. 00000000E00	. 18393972E00
- . 31606028E00	- . 86787944E00
. 10000000E01	

Figure A-7. Data for example 3.

The computer results may be checked by considering the expression in the form

$$G = \frac{(Z-0.5) (Z-0.333) (30) (1-e^{-TS})}{Z (Z+0.5) (S^2) (S+10)} ,$$

which transforms into

$$G(Z) = \frac{(Z-0.5) (Z-0.333) (1.104) (Z + -0.718)}{Z (Z+0.5) (Z-1) (Z-0.368)} ,$$

and when multiplied out becomes:

$$G(Z) = \frac{1.104 Z^3 - 0.127 Z^2 - 0.476 Z + 0.132}{Z^4 - 0.868 Z^3 - 0.3160 Z^2 + 0.1840 Z}$$

This checks with the computer results given in Figure A-7.

When rounding off the computer results it is observed that they compare with the calculated values.

$$\frac{0.132 - 0.477 Z - 0.127 Z^2 + 1.104 Z^3}{0.184 Z - 0.316 Z^2 - 0.868 Z^3 + Z^4} = G(Z)$$

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APPROVAL

TMX-53866

Z TRANSFORM AND THE USE OF THE DIGITAL DIFFERENTIAL
ANALYZER AS A PERIPHERAL DEVICE TO A
GENERAL PURPOSE COMPUTER

By Donald Nalley

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

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